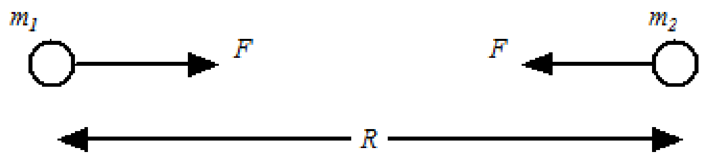
# Gravitation

The magnitude of the gravitational force between two objects of masses $m_1$ and $m_2$ seperated by a distance $R$ as shown below is given by the equation




$$F = \frac{Gm_1m_2}{R^2}$$

The force of gravity is always atractive and is directed along the line joining the two masses. The object of mass $m_1$ experiences a force of magnitude $F$ pointing towards the object of mass $m_2$ and the object of mass $m_2$ experiences a force of magnitude $F$ pointing towards the object of mass $m_1$ as shown in the figure above. The constant $G$ in the above equation is called the Universal constant of gravitation and it has a value of $6.67 \times 10^{-11}\ N\ m^2/kg^2$.

### Example

Let us determine the force of gravity between two 60 $kg$ objects seperated by a distance of 1 $m$.

$$\begin{aligned} F &= \frac{Gm_1m_2}{R^2} \\ &= \frac{(6.67 \times 10^{-11})(60)(60)}{1^2} \\ &= 2.401 \times 10^{-7}\ N \end{aligned}$$

### Problems

1. Determine the force of gravity between two 80 $kg$ objects seperated by a distance of 2 $m$.
2. Determine the force that the Earth exerts on a 10 $kg$ object on its surface. The distance between the center of the Earth and the object is $6.38 \times 10^6\ m$. The mass of the Earth is $5.98 \times 10^{24}\ kg$. What is the weight of the object?
3. Determine the force that the Sun exerts on the Earth. Mass of the Sun is $1.99 \times 10^{30}\ kg$, Mass of the Earth is $5.98 \times 10^{24}\ kg$, the distance between the Sun and the Earth is $1.496 \times 10^{11}\ m$.
4. Determine the gravitational force between an electron and a proton in the hydrogen atom. The mass of the electron is $9.11 \times 10^{-31}\ kg$, mass of the proton is $1.67 \times 10^{-27}\ kg$ and the average distance between the two is $5.3 \times 10^{-11}\ m$.
5. The force of gravity between two objects of masses 1000 $kg$ is measured to be $5 \times 10^{-4}\ N$. Determine the distance between the masses.

# Acceleration due to gravity

In the previous section we learnt how to find the force of gravity between two objects. There we notice that the magnitude of the force experienced by each of the two masses is the same. Do we expect the acceleration of each mass to be the same? In general the acceleration of the two masses will be different. The only case when the two masses will have the same acceleration is when the two masses are equal in magnitude. Using Newton's Second Law, $F = ma$, we can obtain the acceleration of block of mass $m_1$ using

$$a_1 = \frac{F}{m_1} = \frac{Gm_1m_2}{R^2m_1} = \frac{Gm_2}{R^2}$$

Notice that the acceleration of the block of mass $m_1$ only depends on the mass of the second block and the distance between the two blocks. It does not depend on $m_1$.
We can now write the equation for the acceleration of block $m_2$ as

$$a_2 = \frac{Gm_1}{R^2}$$

### Example

Let us determine the acceleration due to gravity of a 10 $kg$ block on the surface of the Earth. The mass of the Earth is $M_E = 5.98 \times 10^{24}\ kg$, the radius of the Earth is $R = 6.38 \times 10^6\ m$.

$$\begin{aligned} a &= \frac{GM_E}{R^2} \\ &= \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{(6.38 \times 10^6)^2} \\ &= 9.799\ m/s^2 \end{aligned}$$

We have approximated this value to 9.8 $m/s^2$. One should notice that multiplying this number by the mass of the object gives us the weight of the object, which is nothing but the Force of Gravity on the object due to the EARTH.

### Problems

1. Determine the acceleration due to gravity of object of mass 100 $kg$ that is at a distance of $1.276 \times 10^7$ $m$ from the center of the earth. What is the weight of this object at this location? What would be the weight of this object near the surface of the Earth?

2. The mass of Jupiter is $1.90 \times 10^{27}\ kg$ and its radius is $6.99 \times 10^7\ m$. Determine the acceleration due to gravity near the surface of Jupiter. Compare it with the acceleration due to gravity near the Earth's surface. Compare the weight of the object on the two planets.

3. The mass of the Moon is $7.36 \times 10^{22}\ kg$ and its radius is $1.74 \times 10^6$. Determine the acceleration due to gravity near the surface of the moon and compare this value with that for the Earth.

## Motion of Planets and Satellites

The motion of planets around stars or satellites around planets is governed by Kepler's three Laws.
The first law states that each planet travels in an elliptical orbit around the sun, and the sun is at one of the focal points.
The second law states that an imaginary line draw from the sun to any planet sweeps out equal areas in equal time intervals.
The third law states that the square of the planet's orbital period ($T^2$) is proportional to the cube of the average distance ($r^3$) between the planet and the sun, or $T^2 \propto r^3$.
In what is to follow we will only consider circular orbits. Let us first obtain the speed with which a planet travels around its star in a circular orbit. Consider a planet of mass $M_p$ that is moving in a circular orbit of radius $R$ around a star of mass $M_S$. We know that the force of gravity between the two is equal to the

centripetal force experienced by the planet. Let us equate these two forces and obtain the speed of the planet.

$$\begin{aligned} F_g &= F_c \\ \frac{GM_S M_P}{R^2} &= \frac{M_P v^2}{R} \\ \frac{GM_S}{R^2} &= \frac{v^2}{R} \\ v^2 &= \frac{GM_S}{R} \\ v &= \sqrt{\frac{GM_S}{R}} \end{aligned}$$

Notice that the speed is independent of the mass of the planet and only depends on the mass of the star and the radius of the orbit. In the case of a satellite revolving around a planet, the speed of the satellite will be dependent on the mass of the planet and the radius of the path given by

$$v = \sqrt{\frac{GM_p}{R}}$$

## Example

A satellite is in a circular orbit around the Earth ($M_E = 5.98 \times 10^{24}\ kg$). The radius of the orbit is $7.2 \times 10^6$ $m$. The speed of the satellite is given by

$$\begin{aligned} v &= \sqrt{\frac{GM_E}{R}} \\ &= \sqrt{\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{7.2 \times 10^6}} \\ &= 7443\ m/s \end{aligned}$$

Determine the time period of this satellite.

## Problems

1. A satellite is placed in a circular orbit around the earth. The radius of the orbit is $1.2 \times 10^7\ m$. Determine the speed and the period of the satellite.

2. A satellite in a circular orbit around the earth has a speed of 8000 $m/s$. Determine the radius of its orbit. Determine the time period.

3. Planet X is observed to travel around star S with a speed of 30000 $m/s$ in a circular orbit of radius $2 \times 10^{11}\ m$. Determine the mass of the star. Determine the period of revolution for the planet.

## Kepler's Third Law

Let us now derive Kepler's Third Law ($T^2 \propto R^3$) for a circular orbit. We can obtain this relation by equating the two expressions for the velocity in a circular orbit, namely

$$v = \frac{2\pi R}{T}$$

and

$$v = \sqrt{\frac{GM}{R}}$$

Using the two expressions, we have

$$\begin{aligned} \frac{2\pi R}{T} &= \sqrt{\frac{GM}{R}} \\ \frac{4\pi^2 R^2}{T^2} &= \frac{GM}{R} \\ (4\pi^2 R^2)R &= (GM)T^2 \\ \frac{4\pi^2 R^3}{GM} &= T^2 \end{aligned}$$

We notice that for a particular Star-Planet (Planet-Satellite) system the mass of the central object (star or planet) and $G$ can be considered constants. We therefore notice that $T^2 \propto R^3$ which is the statement of Kepler's Third Law.

## Example

A satellite is in a circular orbit of radius $8 \times 10^7$ $m$ around the Earth. Determine the period of its motion. Using the above expression we get

$$\begin{aligned} T^2 &= \frac{(4)(\pi^2)(8 \times 10^7)^3}{(6.67 \times 10^{-11})(5.98 \times 10^{24})} \\ T^2 &= 5.0676 \times 10^{10} \\ T &= 225113.3\ s \\ &= 2.6\ days \end{aligned}$$

## Problems

1. The radius of the circular orbit of a satellite around the earth is $8 \times 10^6$ $m$. Determine the time period for the motion.

2. A satellite orbits the Earth once every 6 hours. Determine the radius of its orbit.